Finite Element, Large-Deflection Random Response of Thermally Buckled Beams

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The effects of temperature and acoustic loading are included in a theoretical finite element, large-deflection formulation for the random response of thin, isotropic beams. Thermal loads are applied as steady-state temperature distributions, and acoustic loads are taken to be ergodic and Gaussian with zero mean and uniform magnitude and phase along the length of the beam. Material properties are considered to be independent of temperature. Also, in-plane and rotary inertia terms are assumed to be negligible, and all in-plane edge conditions are taken to be immovable. For the random response analysis, both auto- and cross-correlation terms are included. The nature of the loads leads to the solution of two separate problems. First, the problem of thermal postbuckling is solved to determine the deflections and stresses due to the thermal load only. These deflections and stresses are then used as initial deflections and stresses for the random vibration analysis. Root-mean-square maximum deflections and strains are obtained and compared with previous classical equivalent linearization results.

Introduction

MODERN applications of structural mechanics frequently involve the use of high strength, lightweight materials that are designed to endure combinations of severe static and dynamic loadings. These severe loadings can produce nonlinear structural behavior, which has typically been dealt with in an empirical fashion by testing various structural components in simulated loading environments. However, with advanced materials, structures can be tailored for a specific purpose, and since testing is not practical for every conceivable structural configuration, the best design could be one for which no test data are available. To better evaluate potential designs, nonlinear analysis methods have been developed for particular types of structures and loadings, but situations do occur for which general-purpose nonlinear analytical methods are not available, for example, lightweight thin structural components subjected to a combined thermal-acoustic loading.

Current analytical design methods for sonic fatigue[1-5] are based primarily on linear structural theory with empirical corrections to account for nonlinear behavior and are valid only for specific applications. Numerous studies, both classical[6-10] and finite element,[11-13] have demonstrated that analytical predictions for acoustically loaded plates and beams are greatly improved using nonlinear multiple-degree-of-freedom models. Of these studies, only Seide and Adami[10] have considered the response of structures subjected to thermal-acoustic loading. They used the classical Woinowsky-Krieger beam equation to model the random response of thermally buckled beams. It was concluded that as many as 100 modal functions could be necessary for the accurate determination of stresses. The present study is the first finite element formulation for the large-deflection random response of thermally buckled beams. It should greatly aid in the development of improved analytical design methods for aerospace vehicles subjected to a combined thermal-acoustic loading. Results are found to be in excellent agreement with Seide and Adami's 100-mode classical beam solution.[10]

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Mathematical Formulation

The governing nonlinear equations of motion are derived for a beam subjected to a combined thermal-acoustic loading. The thermal loading is considered to be a static preload with corresponding static deflection w_0. To compute the deflection w_0, it is necessary to solve the problems of thermal buckling and postbuckling. The uniform acoustic loading p is then applied to the thermally buckled beam shown in Fig. 1. Note that the random deflection w is measured with respect to the initial static deflection w_0.

Nonlinear Equations of Motion

The von Karman large-deflection strain-displacement relation for an initially deflected beam[14] is given by

$$\epsilon = e + z\kappa \tag{1}$$

where

$$e = e_m + e_b$$

$$e_m = u_{,x}$$

$$e_b = \tfrac{1}{2} w_{,x}^2 + w_{,x} w_{0,x}$$

$$\kappa = -w_{,xx}$$

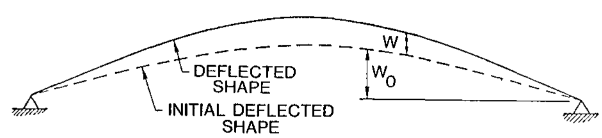

Fig. 1 Thermally buckled beam.

where e is the axial strain, κ is the curvature, u and w are displacements in the x and z directions, respectively, and w_0 is an initial displacement.

For an isotropic Hookean beam subjected to a temperature distribution $\Delta T(x)$, the in-plane force and moment resultants are given by

$$N = EAe + N_0 - EA\alpha\Delta T(x)$$
$$M = EI\kappa + M_0 \quad (2)$$

where N_0 and M_0 are the initial in-plane force and bending moment, respectively. The principle of virtual work states that for a structure in equilibrium under the action of internal and external forces, the work done by these forces in undergoing an infinitesimal virtual displacement is zero. For the present problem, this can be stated as

$$\delta W_{\text{int}} = \delta W_{\text{ext}} \quad (3)$$

where δW_{int} represents the virtual work of the internal forces and δW_{ext} represents the virtual work of the external forces. The virtual work of the internal forces can be written as

$$\delta W_{\text{int}} = \int(\delta e N + \delta\kappa M)\, \mathrm{d}x \quad (4)$$

and the virtual work of the external forces including inertia and damping forces is given by

$$\delta W_{\text{ext}} = \int[\delta w(p - \rho A\ddot{w} - c\dot{w})]\, \mathrm{d}x \quad (5)$$

For displacement functions

$$w = \alpha_1 + \alpha_2 x + \alpha_3 x^2 + \alpha_4 x^3 = [H_w]\{\alpha\}$$
$$u = \beta_1 + \beta_2 x = [H_u]\{\beta\} \quad (6)$$

the generalized coordinates

$$\{\alpha\}^T = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]$$
$$\{\beta\}^T = [\beta_1, \beta_2] \quad (7)$$

can be determined from the nodal displacements

$$\{a\}^T = [\{a_b\}^T, \{a_m\}^T] \quad (8)$$

where

$$\{a_b\}^T = [w_1, \theta_1, w_2, \theta_2]$$
$$\{a_m\}^T = [u_1, u_2]$$

These nodal displacements for the beam element are shown in Fig. 2. The generalized coordinates written in terms of the nodal displacements are given as

$$\{\alpha\} = [T_b]\{a_b\}$$
$$\{\beta\} = [T_m]\{a_m\} \quad (9)$$

where $[T_b]$ and $[T_m]$ are transformation matrices. Explicit expressions for $[T_b]$ and $[T_m]$ can be found in Ref. 15. Combining Eqs. (1), (6), and (9), the element strains can be expressed as

$$e_m = [B_m]\{a_m\}$$
$$e_b = \tfrac{1}{2}\theta[B_\theta]\{a_b\} + \theta_0[B_\theta]\{a_b\}$$
$$\kappa = [B_b]\{a_b\} \quad (10)$$

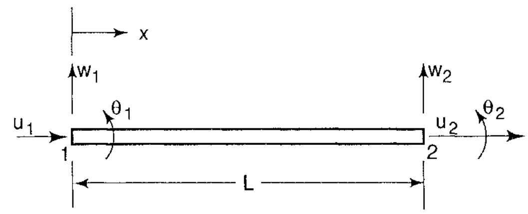

Fig. 2 Beam element nodal displacements.

where

$$\theta = w_{,x} = [B_\theta]\{a_b\}$$
$$\theta_0 = w_{0,x} = [B_\theta]\{a_{b0}\}$$

and $\{a_{b0}\}$ are the initial nodal displacements. Using Eqs. (10), the virtual strains become

$$\delta e_m = [B_m]\{\delta a_m\}$$
$$\delta e_b = \theta[B_\theta]\{\delta a_b\} + \theta_0[B_\theta]\{\delta a_b\}$$
$$\delta\kappa = [B_b]\{\delta a_b\} \quad (11)$$

Combining Eqs. (1–6), (9), (10), and (11), the element equations of motion can be written in the form

$$[k + \tfrac{1}{2}n_1 + \tfrac{1}{3}n_2]\{a\} + [c]\{\dot{a}\} + [m]\{\ddot{a}\} = \{p\}$$

with corresponding system equations

$$[K + \tfrac{1}{2}N_1 + \tfrac{1}{3}N_2]\{Q\} + [C]\{\dot{Q}\} + [M]\{\ddot{Q}\} = \{P\} \quad (12)$$

where $[n_1]$, $[n_2]$, $[N_1]$, and $[N_2]$ represent the element and system first- and second-order nonlinear matrices, respectively. A complete description of these matrices is given by Locke.[15]

Thermal Postbuckling

For the thermal postbuckling of a beam that is initially flat and unstressed, the equations of motion can be obtained from Eq. (12) by taking w_0, N_0, and M_0 equal to zero and by neglecting the dynamic terms. The resulting equations of motion take the form

$$[K + \tfrac{1}{2}N_1 + \tfrac{1}{3}N_2]\{Q\} = \{P\} \quad (13)$$

Equation (13) can be solved for a given thermal loading by using the method of Newton-Raphson iteration, which for the present problem can be written as

$$[K + N_1 + N_2]_i\{\Delta Q\}_{i+1} = \{\Delta P\}_i \quad (14)$$

where

$$\{\Delta P\}_i = \{P\} - [K + \tfrac{1}{2}N_1 + \tfrac{1}{3}N_2]_i\{Q\}_i$$
$$\{Q\}_{i+1} = \{Q\}_i + \{\Delta Q\}_{i+1}$$

For a given iteration i, Eq. (14) can be solved to obtain the incremental displacement $\{\Delta Q\}_{i+1}$, which can then be used to obtain the updated displacement $\{Q\}_{i+1}$. The solution was considered to be converged when either the modified absolute norm or the modified Euclidean norm[16] were less than or equal to 10^{-6}.

Large-Deflection Random Vibration

The equations of motion for large-deflection random vibration can be obtained from Eq. (12) by setting $\Delta T(x)$ equal to zero where w_0 represents the buckled shape with corresponding in-plane force N_0 and moment M_0. Under these conditions the equations of motion take the form

$$\left\{\begin{bmatrix} [K_b] & [K_{mb}]^T \\ [K_{mb}] & [K_m] \end{bmatrix} + \frac{1}{2}\begin{bmatrix} [N1_b] & [N1_{mb}]^T \\ [N1_{mb}] & 0 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} [N2_b] & 0 \\ 0 & 0 \end{bmatrix}\right\}\begin{Bmatrix} \{Q_b\} \\ \{Q_m\} \end{Bmatrix} + \xi\begin{bmatrix} [M_b] & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \{\dot{Q}_b\} \\ 0 \end{Bmatrix} + \begin{bmatrix} [M_b] & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \{\ddot{Q}_b\} \\ 0 \end{Bmatrix} = \begin{Bmatrix} \{P_b\} \\ 0 \end{Bmatrix} \tag{15}$$

where the damping matrix has been taken to be proportional to the mass matrix. If the in-plane displacements are expressed in terms of the bending displacements, the linear frequencies ω_i^2 and mode shapes $\{\phi\}_i$ for the thermally buckled beam are given by

$$[K_b - K_{mb}^T K_m^{-1} K_{mb}]\{\phi\}_i = \omega_i^2 [M_b]\{\phi\}_i \tag{16}$$

Solving Eq. (16), the bending nodal displacements can be written in terms of the mode shapes as

$$\{Q_b\} = [\{\phi\}_1, \ldots, \{\phi\}_n]\{q\} = [\Phi]\{q\} = \sum_{i=1}^{n} q_i\{\phi\}_i \tag{17}$$

Using Eq. (17) all of the nodal displacements can be written in terms of the mode shapes $\{\phi\}_i$ and the modal coordinates q_i, and Eq. (15) can be transformed to a system of nonlinear nodal equations expressed as

$$\left\{[k] + \frac{1}{2}\sum_{j=1}^{n} q_j [k1]_j + \sum_{j=1}^{n}\sum_{k=1}^{n} q_j q_k [k2]_{jk}\right\}\{q\} + \xi[m]\{\dot{q}\} + [m]\{\ddot{q}\} = \{f\} \tag{18}$$

where

$$[k] = [\Phi]^T[K_b - K_{mb}^T K_m^{-1} K_{mb}][\Phi] = \begin{bmatrix} \omega_1^2 m_1 & & \\ & \ddots & \\ & & \omega_n^2 m_n \end{bmatrix}$$

$$[k1]_j = [\Phi]^T\{[N1_b]_j - [K_{mb}^T K_m^{-1}][N1_{mb}]_j - [N1_{mb}]_j^T[K_m^{-1}K_{mb}]\}[\Phi]$$

$$[k2]_{jk} = [\Phi]^T\{ \tfrac{1}{3}[N2_b]_{jk} + \tfrac{1}{4}[N1_b]_{jk} - \tfrac{1}{4}[N1_{mb}]_j^T[K_m]^{-1}[N1_{mb}]_k\}[\Phi]$$

$$[m] = \begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{bmatrix}$$

$$\{f\} = [\Phi]^T\{P_b\}$$

Consider that Eq. (18) can be written in the form

$$\{g(q)\} + \xi[m]\{\dot{q}\} + [m]\{\ddot{q}\} = \{f\}$$

where

$$\{g(q)\} = \left\{[k] + \frac{1}{2}\sum_{j=1}^{n} q_j [k1]_j + \sum_{j=1}^{n}\sum_{k=1}^{n} q_j q_k [k2]_{jk}\right\}\{q\} \tag{19}$$

If Eq. (19) were linear, it could be expressed in the form

$$[\bar{k}]\{q\} + \xi[m]\{\dot{q}\} + [m]\{\ddot{q}\} = \{f\} \tag{20}$$

where $[\bar{k}]$ is an equivalent linear stiffness matrix. The error involved in using Eq. (20) instead of Eq. (19) is given by the difference between the two equations as

$$\{e\} = \{g(q)\} - [\bar{k}]\{q\} \tag{21}$$

The equivalent linear stiffness can be found by requiring that the mean-square value of $\{e\}$ be a minimum, that is

$$E[\{e\}^T\{e\}] \rightarrow \text{minimum} \tag{22}$$

Applying Eq. (22), the equivalent linear stiffness can be determined from the equation[17]

$$E[\{q\}\{q\}^T]\,[\bar{k}] = E[\{q\}\{g(q)\}^T] \tag{23}$$

If the covariance matrix $E[\{q\}\{q\}^T]$ is known, then Eq. (23) can be used to determine the equivalent linear stiffness $[\bar{k}]$. Conversely, if $[\bar{k}]$ is known, then Eq. (20) can be used to determine the covariance matrix. The equivalent linear frequencies and mode shapes for the system described by Eq. (20) can be determined from the equation

$$[\bar{k}]\{\tilde{\phi}\} = \Omega^2[m]\{\tilde{\phi}\} \tag{24}$$

Applying the coordinate transformation

$$\{q\} = [\tilde{\Phi}]\{\eta\} \tag{25}$$

where

$$[\tilde{\Phi}] = [\{\tilde{\phi}\}_1, \ldots, \{\tilde{\phi}\}_n]$$

Equation (20) becomes uncoupled, yielding the modal equations

$$\ddot{\eta}_j + \xi\dot{\eta}_j + \Omega_j^2\eta_j = \tilde{f}_j \tag{26}$$

where

$$\tilde{f}_j = \frac{\{\tilde{\phi}\}_j^T\{f\}_j}{\tilde{m}_j}$$

$$\tilde{m}_j = \{\tilde{\phi}\}_j^T[m]\{\tilde{\phi}\}_j$$

For the system described by Eq. (26), the covariance terms for the case of ideal white noise are given by[18]

$$E[\eta_j\eta_k] = S_0\tilde{f}_j\tilde{f}_k I_{jk} \tag{27}$$

where

$$I_{jk} = \int_{-\infty}^{\infty} H_j(\omega)H_k(-\omega)\,\mathrm{d}\omega$$

$$H_j(\omega) = \frac{1}{\Omega_j^2 - \omega^2 + i\xi\omega}$$

and S_0 is the double-sided spectral density of the uniformly applied external loading in units of (force/length)^2/rad/s.

Using the Residue Theorem, I_{jk} is found to be

$$I_{jk} = \frac{4\pi\xi}{(\Omega_k^2 - \Omega_j^2)^2 + 2\xi^2(\Omega_j^2 + \Omega_k^2)} \tag{28}$$

Typically, the spectral density of the excitation is single-sided and is given in terms of (force/length)^2/Hz; Eq. (27) modified for this case becomes

$$E[\eta_j\eta_k] = S_p\tilde{f}_j\tilde{f}_k\left[\frac{\xi}{(\Omega_k^2 - \Omega_j^2)^2 + 2\xi^2(\Omega_j^2 + \Omega_k^2)}\right] \tag{29}$$

where S_p is the single-sided spectral density of the uniformly applied external loading, $p = p_0$ for Eq. (5), in units of (force/length)^2/Hz, and the nodal forces $\{P_b\}$ of Eq. (15) correspond to $p_0 = 1$.

Using Eq. (25), the covariance matrix $E[\{q\}\{q\}^T]$ becomes

$$E[\{q\}\{q\}^T] = E\{[\tilde{\Phi}]\{\eta\}\{\eta\}^T[\tilde{\Phi}]^T\} = [\tilde{\Phi}]E[\{\eta\}\{\eta\}^T][\tilde{\Phi}]^T \quad (30)$$

where the terms for the covariance matrix $E[\{\eta\}\{\eta\}^T]$ are given by Eq. (29). Therefore, Eqs. (23), (24), (26), (29), and (30) can be used to determine $[\bar{k}]$ and $E[\{q\}\{q\}^T]$. However, since each of these quantities is dependent on the other, these equations are nonlinear and, consequently, must be solved using an iterative method. As a first approximation, consider neglecting all cross-correlation terms, i.e., $E[q_i q_j] = 0$ for $i \neq j$, and assuming that all of the equations are completely uncoupled. For this case, the diagonal terms in the equivalent linear stiffness matrix can be computed from Eq. (23) as

$$\bar{k}_{jj} = k_j + 3\, k2_{jjjj}E[q_j^2] \quad (31)$$

where k_j and $k2_{jjjj}$ can be determined from Eq. (18) Alternatively, Eq. (31) can be written as

$$\Omega_j^2 = \frac{\bar{k}_{jj}}{m_j} = \omega_j^2 + 3\, k2_{jjjj}E[q_j^2]/m_j \quad (32)$$

Since all of the modes are assumed to be uncoupled, the equivalent linear stiffness matrix $[\bar{k}]$ is diagonal and $\eta_j = q_j$. As a result of this, $\bar{f}_j = f_j/m_j$, and $E[q_j^2]$ can be found from Eq. (29) to be

$$E[q_j^2] = S_p \frac{f_j^2}{m_j^2}\left(\frac{1}{4\xi\Omega_j^2}\right) \quad (33)$$

Using Eqs. (32) and (33), $E[q_j^2]$ can be determined to be

$$E[q_j^2] = (\sqrt{B^2 + 4C} - B)/2 \quad (34)$$

where

$$B = \frac{k_j}{3\, k2_{jjjj}}$$

$$C = \frac{S_p f_j^2}{12\xi m_j\, k2_{jjjj}}$$

The covariance matrix of the displacements $\{Q_b\}$ can be found using Eq. (17) which yields

$$E[\{Q_b\}\{Q_b\}^T] = [\Phi]E[\{q\}\{q\}^T][\Phi]^T \quad (35)$$

To begin the coupled solution, the cross-correlation terms $E[q_i q_j]$ for the covariance matrix can be evaluated using Eq. (29) with the equivalent linear frequencies given by Eq. (32). Using Eq. (23), the equivalent linear stiffness can be computed. Equations (24), (26), (29), and (30) can then be used to determine a new covariance matrix, and Eq. (35) can be used to compute the nodal displacement covariance matrix. If the convergence criteria

$$\left|\frac{(\Delta w_{\text{rms}})_i}{(w_{\text{rms}})_i}\right| \leq 10^{-5}$$

are satisfied, then the iterative process is terminated. Here, i is the iteration, $(\Delta w_{\text{rms}})_i$ is the change of the center root-mean-square (RMS) deflection from the previous iteration, and $(w_{\text{rms}})_i$ is the center RMS deflection for the current iteration.

This direct iteration method can be used to determine the mean-square response; however, it is slow to converge. An improved method to use for hardening-type structures[19] is an under-relaxation approach where displacements are not updated to full values after an iteration. For the present problem, this method can be used for the covariance matrix as

$$E[\{q\}\{q\}^T]_{i+1} = (1-\beta)E[\{q\}\{q\}^T]_i + \beta E[\{q\}\{q\}^T]_{i+1} \quad (36)$$

where $E[\{q\}\{q\}^T]_{i+1}$ is the computed covariance matrix for the current iteration and $E[\{q\}\{q\}^T]_i$ is for the previous iteration. This method was found to work very well with $\beta = 0.5$.

RMS Strains

Using Eq. (1), the extreme fiber strain for a given displacement can be determined to be

$$\epsilon_{\pm h/2} = e_m + e_b \pm (h/2)\kappa \quad (37)$$

For this particular element, the membrane strain e_m due to in-plane displacements and the curvature κ are discontinuous. Following Oden and Brauchli,[20] these can be transformed to continuous quantities denoted e_{mc} and κ_c by assuming that the element displacement functions given by Eq. (6) can also be used to describe the strains and curvatures. Hence, Eq. (37) becomes

$$\epsilon_{\pm h/2} = e_{mc} + e_b \pm (h/2)\kappa_c \quad (38)$$

Equation (38) can be written in the form

$$\epsilon_{\pm h/2} = \sum_{j=1}^{n}[e_{mc} + e_b \pm (h/2)\kappa_c]_j\, q_j + \sum_{j=1}^{n}\sum_{k=1}^{n}(e_{mc} + e_b)_{jk} q_j q_k \quad (39)$$

Using Eq. (39), the mean-square strain can be evaluated as

$$E[(\epsilon_{\pm h/2})^2] = \sum_{i=1}^{n}\sum_{j=1}^{n}\{[e_{mc} + e_b \pm (h/2)\kappa_c]_i [e_{mc} + e_b \pm (h/2)\kappa_c]_j\, E[q_i q_j]\} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\ell=1}^{n}\{(e_{mc} + e_b)_{ij}(e_{mc} + e_b)_{k\ell}\, E[q_i\, q_j\, q_k\, q_\ell]\} \quad (40)$$

and Eq. (40) can be used to determine the RMS strain. A more complete description of the computational procedures is given by Locke.[15]

Results and Discussion

The primary objective of this study was to develop a finite element formulation for the large-deflection random response of thermally buckled beams. To verify the finite element formulation, results were compared with previous analytical studies[9,10] for both flat, unstressed beams and a simply supported beam buckled by a uniform temperature distribution. All in-plane edges are taken to be immovable. The damping ξ is taken to be equal to $2\zeta\omega_0$ where ζ is the damping ratio and ω_0 is the fundamental frequency of the initially flat, stress-free beam. The material properties, mass density, and damping ratio are taken as 1) Young's modulus, $E = 10.5 \times 10^6$ psi; 2) Poisson's ratio, $\nu = 0.3$; 3) coefficient of thermal expansion $\alpha = 12.5 \times 10^{-6}$ in./in./°F; 4) mass density, $\rho = 0.2588 \times 10^{-3}$ lb $-$ s^2/in.^{4}; and 5) damping ratio, $\zeta = 0.01$ with the dimensions of the beam being length, $a = 12$ in.; width, $b = 2$ in.; and thickness, $h = 0.064$ in.

A 16-element half-beam model is used to evaluate the convergence characteristics of the present formulation and to determine the required number of modes for reasonable accuracy. The half-beam model was chosen since both the thermal and random loading are considered to be symmetric. As a result, only the symmetric mode shapes are used for the random vibration analysis. Sixteen elements were chosen since a high degree of mesh refinement is necessary to accurately determine the higher frequencies and mode shapes used for both the convergence study and subsequent analyses.

The RMS maximum strains obtained using one, two, three, and four symmetric mode shapes for flat, unstressed beams are shown in Figs. 3 and 4. Convergence is very rapid, and comparisons with previous 100-mode classical results[9,10] are excellent as demonstrated in Figs. 5-8. For these figures, T/T_{cr} is the ratio of the actual temperature to the critical buckling temperature, and w/r is the ratio of the beam deflection w to the radius of gyration r of the beam cross section. Discrepancies can be attributed to the low-order of the element in-plane displacement function as well as the fact that the classical solutions use such a large number of modes. Furthermore, the method of equivalent linearization used for the present study does not make the restrictive assumption that the equivalent linear stiffness matrix is diagonal, which, in general, is not true.

Having verified the accuracy of the present formulation, the RMS strain response at the center of a thermally buckled, simply supported beam was computed at various sound spectrum levels for two cases: a uniform temperature distribution and a nonuniform temperature distribution. For the case of a uniform temperature distribution $\Delta T(x) = T_0$, the temperature T_0 necessary to produce buckling was found to be 1.87°F, whereas for the nonuniform temperature distribution $\Delta T(x) = T_0 \sin(\pi x/a)$, the temperature T_0 to produce buckling was found to be 2.94°F. Although the maximum temperature at the center of the beam is higher for the nonuniform temperature distribution, the average temperature is the same ($T_{AVG} = 1.87$°F) for both cases. The results for the two different cases are shown in Figs. 9 and 10. As illustrated at low-sound spectrum levels, the maximum strain occurs at or near T/T_{cr} equal to one. However, at higher sound spectrum levels, the maximum strain is seen to occur at higher values of T/T_{cr}. This indicates that even though the beams become stiffer in the postbuckling region, the maximum strains occur at temperatures well beyond the buckling temperature for sufficiently high-sound spectrum levels. This type of strain response is due to the coupling terms of Eq. (1) that depend on both the initial deflection w_0 as well as the random deflection w.

The present study does not consider the dynamic stability of the buckled beams. Since the response is assumed to be Gaussian, the terms related to the stability of the beams [i.e., $q_j[k1]_j$ in Eq. (18)] are not included in the equivalent linear stiffness matrix $[\bar{k}]$. As a result, $[\bar{k}]$ depends quadratically on the displacement, and the equivalent linearized beam is stable. Seide and Adami[10] numerically integrated the equations of motion for thermally buckled beams subjected to random

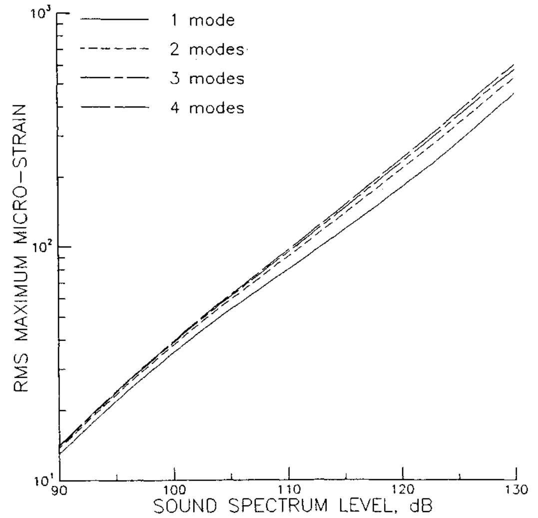

Fig. 3 Convergence of RMS maximum microstrain for a flat, stress-free simply supported beam.

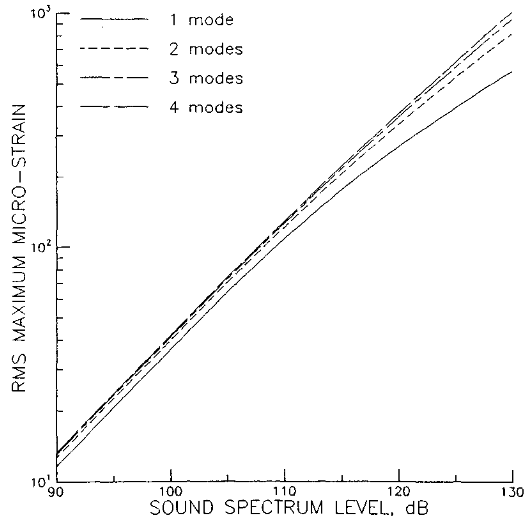

Fig. 4 Convergence of RMS maximum microstrain for a flat, stress-free clamped beam.

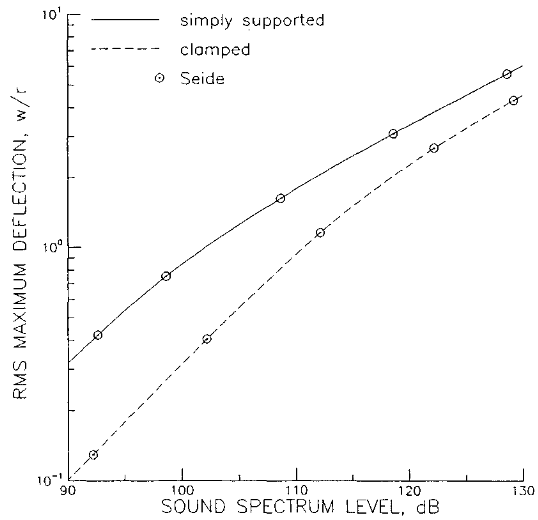

Fig. 5 RMS maximum deflection for flat, stress-free beams.

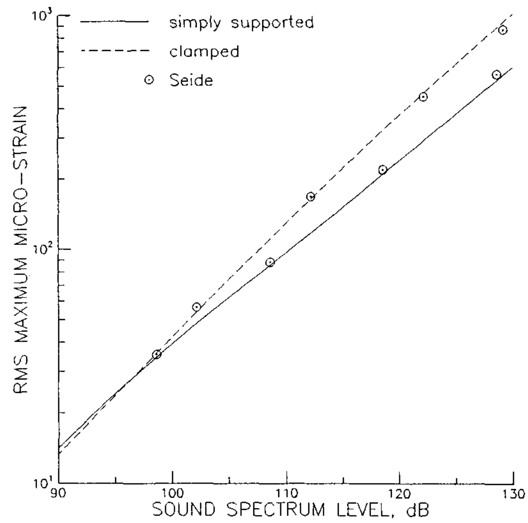

Fig. 6 RMS maximum microstrain for flat, stress-free beams.

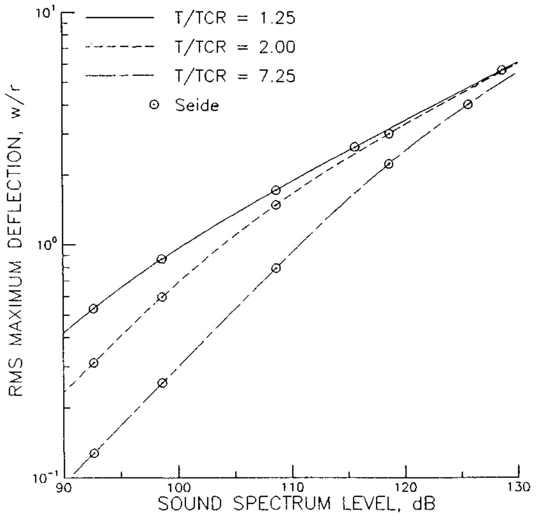

Fig. 7 RMS maximum deflection for a simply supported beam subjected to a uniform temperature distribution.

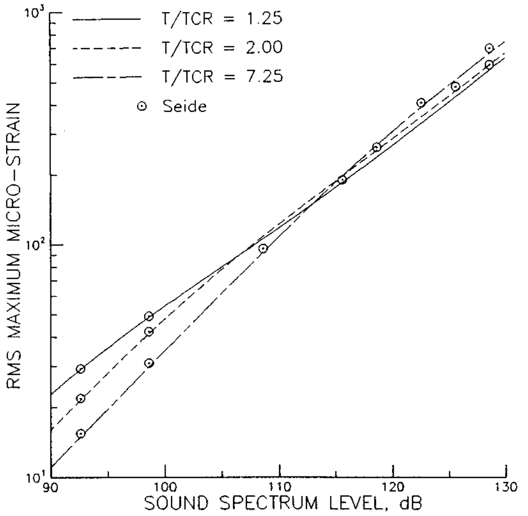

Fig. 8 RMS maximum microstrain for a simply supported beam subjected to a uniform temperature distribution.

excitation and compared the results with equivalent linearization (EL) results. They concluded that the EL method could be used to reasonably bracket the numerical integration results (which include unstable motion). To bracket the results, they used two different EL solutions. It is also mentioned that the numerical integration results could be in error. Thus, it is impossible to conclude how accurate the present EL results are without further research.

The most significant contributions of the present study are considered to be the formulation and solution of the nonlinear modal equations used to describe the large-deflection random response. These general modal equations are applicable not only to the present research but also to other problems involving nonlinear dynamic response, and the present methodology can be extended to built-up structures with complex boundary conditions. For the formulation of the nonlinear modal equations, the linear mode shapes of the thermally buckled structure were used. Previous classical solutions have only considered the mode shapes of the initially flat structure. Therefore, the present formulation should more accurately reflect the dynamic behavior of the thermally buckled structure.

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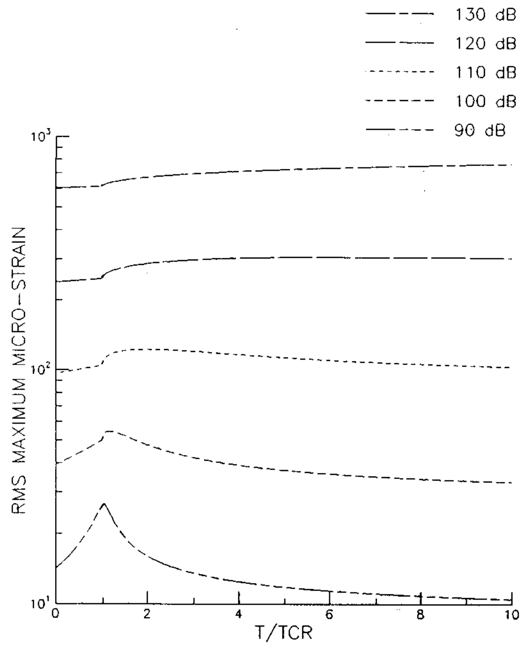

Fig. 9 RMS maximum microstrain vs temperature for a simply supported beam subjected to a uniform temperature distribution.

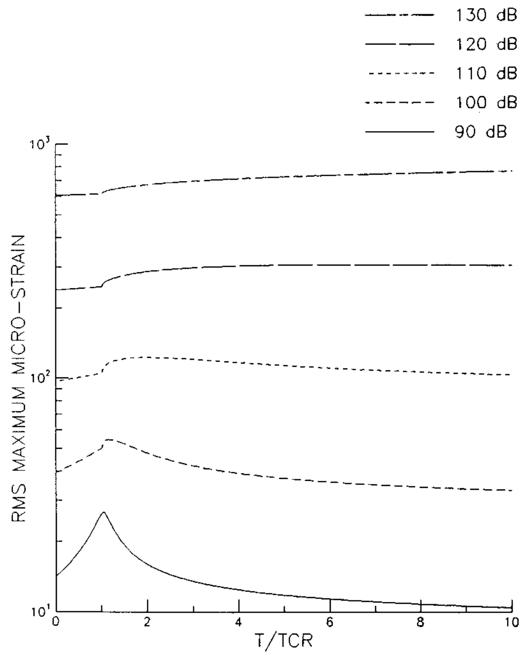

Fig. 10 RMS maximum microstrain vs temperature for a simply supported beam subjected to a nonuniform temperature distribution.

References

[1]Holehouse, I., "Sonic Fatigue Design Techniques for Advanced Composite Aircraft Structures," Wright-Patterson Air Force Base, Ohio, AFWAL-TR-80-3019, April 1980; also Ph.D. Dissertation, Univ. of Southampton, Southampton, England, 1984.

[2]Jacobs, L. D., and Lagerquist, D. R., "Finite Element Analysis of Complex Panel to Random Loads," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-68-44, Oct. 1968.

[3]Rudder, F. F., Jr., and Plumblee, H. E., Jr., "Sonic Fatigue Design Guide for Military Aircraft," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-74-112, May 1975.

[4]Thomson, A. G. R., and Lambert, R. F., "Acoustic Fatigue De-

sign Data," NATO Advisory Group for Aeronautics Research and Development, AGARD-AG-162, Pt. I and II, 1972.

[5]Schneider, C. W., "Acoustic Fatigue Design Criteria for Elevated Temperatures," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-73-155, Pt. II, March 1974.

[6]Prasad, C. B., and Mei, C., "Multiple Mode Large Deflection Random Response of Beams with Nonlinear Damping Subjected to Acoustic Excitation," AIAA Paper 87-2712-CP, Oct. 1987.

[7]Prasad, C. B., "The Effect of Nonlinear Damping on the Large Deflection Response of Structures Subjected to Random Excitation," Ph.D. Dissertation, Old Dominion Univ., Norfolk, VA, 1987.

[8]Mei, C., "Large Deflection Multimode Response of Clamped Rectangular Panels to Acoustic Excitation," Wright-Patterson Air Force Base, Ohio, AFWAL-TR-83-3121, Vol. I, Dec. 1983.

[9]Seide, P., "Nonlinear Stresses and Deflections of Beams Subjected to Random Time Dependent Uniform Pressure," *American Society of Mechanical Engineers Journal of Engineering for Industry*, Vol. 98, No. 3, Aug. 1976, pp. 1014-1020.

[10]Seide, P., and Adami, C., "Dynamic Stability of Beams in a Combined Thermal-Acoustic Environment," Wright-Patterson Air Force Base, Ohio, AFWAL-TR-83-3072, Oct. 1983.

[11]Busby, H. R., and Weingarten, V. I., "Response of Nonlinear Beam to Random Excitation," *ASCE Journal of Engineering Mechanics*, Vol. 99, No. EM1, 1973, pp. 55-68.

[12]Chiang, C. K., and Mei, C., "A Finite Element Large Deflection Multiple-Mode Random Response Analysis of Beams Subjected to Acoustic Loading," *Third International Conference on Recent Advances in Structural Dynamics*, Univ. of Southampton, Southampton, England, July 1988, pp. 769-779.

[13]Chen, C. T., and Yang, T. Y., "Random Vibrations of Geometrically Nonlinear Finite Element Structures," *Proceedings of 5th ASCE Specialty Conference*, Virginia Tech, May 1988, pp. 281-284.

[14]Vol'mir, A. S., *Flexible Plates and Shells*, Translated from Russian, Wright-Patterson Air Force Base, Ohio, AFFDL-TR-66-216, 1967, pp. 13-14.

[15]Locke, J. E., "A Finite Element Formulation for the Large Deflection Random Response of Thermally Buckled Structures," Ph.D. Dissertation, Old Dominion Univ., Norfolk, VA, 1988.

[16]Bergan, P. G., and Clough, R. W., "Convergence Criteria for Iterative Processes," *AIAA Journal*, Vol. 10, No. 8, 1972, pp. 1107-1108.

[17]Atalik, T. S., and Utku, S., "Stochastic Linearization of Multi-Degree-of-Freedom Nonlinear Systems," *Earthquake Engineering and Structural Dynamics*, Vol. 4, 1976, pp. 411-420.

[18]Yang, C. Y., *Random Vibration of Structures*, Wiley, New York, 1986.

[19]Cook, R. D., *Concepts and Applications of Finite Element Analysis*, Wiley, New York, 1981.

[20]Oden, J. T., and Brauchli, H. J., "On the Calculation of Consistent Stress Distributions in Finite Element Approximations," *International Journal for Numerical Methods in Engineering*, Vol. 3, 1971, pp. 317-325.